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Synchronization of Open Systems: Application to the Lossy Power Grid

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Abstract – *The transient stability assessment of multi-machine power systems has been a deeply explored research area in the power system community. The nonlinear nature of the evolving dynamics warrants distinctive control approaches, of which, the time domain simulations and Lyapunov based methods have been majorly applied for assessment and estimating the regions of attraction. However, these methods find limitations when applied to the lossy power grid and demand different modeling procedures. Treating the lossy grid as an open system aids in addressing the challenging problems of assessing their stability and achieving control. In this context, this paper explores the synchronization of open systems through the first order non-uniform Kuramoto model, which renders equivalence between rotor angle stability and phase synchronization of coupled oscillators. This paper investigates the distributed and decentralized control architectures for lossy networks and highlights the contexts under which they are applicable. The mean field Kuramoto model is central to the application of distributed control, whereas a generic framework for developing potential functions using notions of passivity to exercise decentralized control is also discussed.* **Copyright © 2018 Praise Worthy PrizeS.r.l. - All rights reserved.**

***Keywords**: Integrability, Kuramoto Model, Open Systems, Power Grid, Stability, Synchronization*

Nomenclature

δ_j, E_j, E_i	Rotor angle of machine j and voltages at buses i and j
$Y_{ij} = G_{ij} + jB_{ij}$	The admittance matrix, connecting the generators, G_{ij} is the transfer conductance, B_{ij} is the susceptance, $\varphi_{ij} = \tan^{-1}\left(\frac{B_{ij}}{G_{ij}}\right)$, $\beta_{ij} = \frac{\pi}{2} - \varphi_{ij}$
$P_{kj} = \lvert E_k\rvert\lvert E_j\rvert\lvert Y_{kj}\rvert$	Line flows between node k and j
θ_j, Ω_j	Phase and intrinsic frequency of j^{th} oscillator
K	Uniform coupling strength between oscillators
K_{cr}, K_{lock}	Critical coupling strength, coupling strength needed for phase cohesiveness
r, ψ	Magnitude and phase of complex order parameter
$f_d(x), f_I(x), f_{nd}(x)$	Dissipative, invariant and non-dissipative constituents of $f(x)$

I. Introduction

Systems which comprise of parts that affect one another and interact with its environment to form a larger pattern that is different from any of the parts are categorized as open systems.

The system receives information, which it uses to interact dynamically with its environment, thereby, increasing its likelihood to survive and prosper. The definition of an open system assumes that the constituent parts have supplies of energy that are everlasting which are transformed into oscillatory motion. Examples that fall under the class of open systems are the multi-agent systems or many body systems, wherein, different agents or bodies interact with each other resulting in dynamics with emergent patterns. Assessing stability of such systems and applying control through Lyapunov methods using microscopic state variables are centered on the system trajectories reaching an equilibrium point.

However, evaluating stability of open systems through conventional Lyapunov methods proves difficult as they are non-integrable (NI). Ideas of synchronization or consensus are more appropriate to appreciate the stability of such systems. Consensus is said to be achieved, when the equilibrium point is not decided a priory, but the agents agree and converge at a common value, then consensus is achieved. The associated algorithms explain a wide range of multivehicle coordination applications [1] such as rendezvous, formation stabilization, formation maneuvering and flocking, synchronization etc. Synchronous behaviour of slightly interacting units is a ubiquitous phenomenon and the study of synchronization has become a main field of research in nonlinear sciences. These interacting units are modeled as self-sustained oscillators as they are very close mathematical models of natural systems, having features of the ability to produce sustained oscillations despite the dissipations inherent in the system [2]. Ensembles of such oscillators are converted to a network of coupled first order phase oscillators which capture dynamics at critical regimes. The most popular canonical model was proposed by Kuramoto which is used for synchronization of phases of self-sustained oscillators. The model is formulated based on assumptions of weak interactions and diffusive coupling. Each oscillator is characterized by its phase θ_j and the effect of the other oscillators on a given oscillator is governed by the interaction strength.

An oscillator can sustain in its orbit only when the other oscillators interact weakly, thereby enabling it to maintain its motion along the limit cycle, an idea similar to the many body problem discussed further.

Synchronization is achieved through the mechanism of distributed averaging, wherein each oscillator estimates its distance with its nearest neighbour and tries to reduce it. Forming set-valued Lyapunov functions would aid in analyzing the stability of such systems and the existence of a stable equilibrium point is explained through the Brouwer's fixed point theorem. This requires that each agent should move into the convex hull of the other agents, i.e. the states should eventually converge into a convex, compact and a positive invariant (CCPI) set S. If trajectories are converging as per contraction principle, then imposing convexity on the consensus protocol will ensure synchronization, as Brouwer's theorem becomes valid. This paper discusses the conditions that the control law should satisfy to achieve synchronization of rotor angles of an ensemble of N-machines. In the context of a power grid, though the transmission network resistance can be fairly ignored, the load buses which are modeled as constant impedances can be eliminated, and after reduction, the resistive part of the loads appears as transfer conductance (TC) between the rest of the busses and therefore, is not negligible. Such networks are treated as lossy and for a multi-machine power system (MMPS), study of their stability through conventional methods of Lyapunov has been an unrelenting endeavour. However, lossy systems can be stabilized and an effective procedure to derive control laws through Lyapunov methods for such systems is through the idea of drift vector field decomposition. The notions of passivity and the implications of excess or shortage of passivity aid in the analysis and development of effective control methods, which finds application in excitation control of lossy MMPS. This article majorly contributes in reappraising the tools involved in assessing synchronization of rotor angles and puts forward procedures to develop distributed and decentralized control laws for lossy MMPS. Section II deals with the necessary mathematical background while Section III discusses the stability of NI systems, wherein notions of integrability, passivity and set-valued Lyapunov functions are discussed. Section IV revisits the distributed and decentralized control strategies applied for lossy MMPS. Section V provides the conclusions.

II. Mathematical Preliminaries

The following definitions and notations are used in understanding the details discussed in the paper [3].

1) *Convex, Compact and Positive Invariant set*:

a) As the Lyapunov function is formed on the set, one of the basic requirements of the elements of the set is that they should form a convex set.

b) A set $S^*$of real numbers is said to be compact if every sequence in S has a subsequence that converges to an element again contained in S^*.

c) Given $\dot{x} = f(x), x_0$ is the initial point and ϕ is a real valued function. Let:

$$\varrho = x \in \Re^n \mid \phi(x) = 0$$

ϱ is positive invariant if $x_0 \in \varrho$, implies $x(t, x_0) \in \varrho$. Once the trajectory enters ϱ, it remains there. The set ϱ is called a positive invariant set.

2) *Brouwer's fixed point theorem*: If the convex and compact set S^* is positively invariant for the continuous map $f: S^* \rightarrow S^*$, then it necessarily includes a stationary point for the system. Precisely, there exists $\bar{x} \in S^*$, such that $f(\bar{x}) = \bar{x}$. If any of the assumptions of CCPI are dropped, then the remaining two are not sufficient for claiming the existence of the equilibrium point.

III. Stability of Non-Integrable Systems

The conventional procedure for assessing stability of a nonlinear dynamical system is based on formulating a Lyapunov function (LF) V, the sign of its time derivative giving the sufficient conditions to assess the stability of the system. In the case of multi-agent systems, a Lyapunov function V_j is formulated for each agent (which is generally energy based) and the stability of the overall system is judged by:

$$V = \sum_{j=1}^{N} V_j \tag{1}$$

The total V would assist in analyzing the conditions for achieving consensus or propose suitable control.

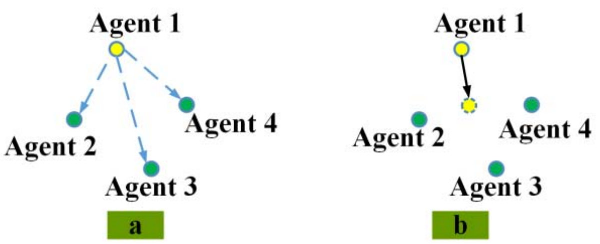
However, forming V as in (1) would fail in scenarios wherein the governing equations of the system are NI.

Set-valued Lyapunov functions help in assessing stability of such systems if synchronization or consensus framework is used. However, formulating V as in (1) is also possible for such systems through passivity approach.

III.1. Rules in Formulating Consensus Protocols

The attainment of consensus would mean that the individual agents communicate with each other and arrive at a value which is in agreement to all of them.

The procedure to achieve this is termed as a consensus protocol which has to adhere to certain conditions so as to achieve coordinated behaviour of agents. The position coordinates of the agents form a boundary, and each agent is influenced by every other agent based on some rule, so that they all move towards a consensus. In the case of a consensus problem, agreement of states involves a distributed averaging phenomenon, which is a memory less diffusive process: at each time instant, each agent receives the current estimates from neighbors and updates its own estimate by a value which lies in the convex hull of all of them [4]. Figs. 1show one iteration of the distributed averaging algorithm of four interacting units. Fig. 1(a) shows that Agent 1 updates its position with respect to the distance between the other three agents, so that it falls within the convex hull of the neighbours, as in Fig. 1(b). Therefore, after four iterations, all the agents move closer and are within the convex hull of their initial values.



Figs. 1. Distributed Averaging Algorithm

Consider a connected graph $\mathcal{G}(\mathcal{V},\mathcal{E})$, with nodes lying in the set $\mathcal{V}$ and edges lying in the set $\mathcal{E}$. Let x_j denote the value of node $v_j \ \forall\, j \in \mathcal{V}$, then, nodes v_j and v_i agree if and only if $x_i = x_j$. If we consider N dimensional states, a function $\Gamma: \Re^n \to \Re$ will result in a single value for the decision variable, or the agreement value: $y = \Gamma(x)$. The function Γ can have one of the following values:

$$\Gamma(x) = \text{Ave}(x) = \frac{1}{N}\sum_{j=1}^{N} x_j \tag{2}$$

$$\Gamma(x) = \text{Max}(x) = \max\{x_1, \dots . x_N\} \tag{3}$$

$$\Gamma(x) = \text{Min}(x) = \min\{x_1, \dots . x_N\} \tag{4}$$

However, there are slight variations to these definitions in the case of synchronization, especially in the case of phase synchronization, where agreement of states implies, $\Omega_i = \Omega_j$ (denoting frequency synchronization) or $\theta_i - \theta_j = 0$ or constant (denoting phase synchronization). The decision variable y would be changing, as each agent would have its own dynamics $\left(f(x_j)\right)$, as well as the influence of the other agents' dynamics(u_j). These interactions result in the generic dynamical model for consensus denoted by:

$$\dot{x}_j = f(x_j) + u_j \text{ where } u_j = \textstyle\sum_i a_{ij}(x_i - x_j) \tag{5}$$

The motive of u_j would be to achieve consensus and the design of the consensus protocol would be based on diffusive coupling. The interconnection matrix a_{ij} has entries of 1 if node i is connected to node j , else has a value of 0. As discussed in [5], a nonlinear consensus update instead of the conventional linear update results in a better convergence rate. This update law would drive the agents to fall within a set S^*, i.e. the resultant value of y should belong to a set S^* which has to satisfy the conditions of CCPI; convexity plays a central role in the convergence of the agents [6].

Checking stability of a system requires constructing an appropriate Lyapunov function, for which prior knowledge of the attractor location is mandatory. In contrast, contraction-based methods demand convergence behavior between neighbouring trajectories. Contraction theory states: A system is stable if in some region any initial conditions or temporary disturbances are somehow forgotten [7]. Contraction theory has been proposed as an effective approach to study convergence between trajectories of a dynamical system [8]. For a system $\dot{x} = f(x,t), x(t_0) = x_0$, where $f(x,t)$ is continuously differentiable, suppose that there exists $\alpha, \xi > 0$, such that any two of its solutions, e. g, $x(t)$, $y(t)$ $\in \Re^n$:

$$|x(t) - y(t)| \leq \alpha e^{-\xi(t-t_0)}|x(t_0) - y(t_0)| \ \forall t > t_0$$

then there exists a Lyapunov function which proves that the system is stable. This notion is called the global exponential stability which is stronger than global asymptotic stability. This is also the principle of contraction which states that any system which is contracting will have a LF.

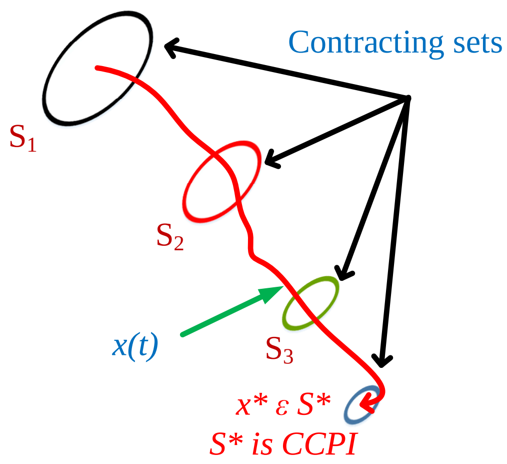


Fig. 2. Contracting sets

The existence of stability of a contracting system can be explained by Brouwer's fixed point theorem. Given an initial state, it would converge into the set S provided the consensus protocol ensures that the trajectories of the system, in the course of their evolution, converge onto a CCPI set. If this is ensured, then Brouwer's theorem states that the set S includes a stationary point for the system. It is to be noted that convexity, boundedness and positive invariance are essential assumptions based on which the Brouwer's theorem manifests. The path taken by the states to converge in S is a contractive phenomenon, i.e. given an initial state; the system evolves into sets which are eventually contractive as in Fig 2.The nonlinear update law exercised in the Kuramoto model depends on the sine of the phase differences between oscillators with intrinsic natural frequencies Ω_j, interacting through an all-to-all network with interconnection strength K:

$$\dot{\theta}_j = \Omega_j + \frac{K}{N}\sum_{i=1}^{N} \sin(\theta_i - \theta_j) \tag{6}$$

If a phase lag (β_{ij}) between the phases is considered, the resulting model becomes non-uniform:

$$\dot{\theta}_j = \Omega_j + \frac{K}{N}\sum_{i=1}^{N} \sin(\theta_i - \theta_j - \beta_{ij}) \tag{7}$$

In the thermodynamic limit, (6) transforms to the mean field Kuramoto model described as:

$$\dot{\theta}_j = \Omega_j + Kr\sin(\psi - \theta_j) \tag{8}$$

where, $z = re^{i\psi} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$ is denoted as the complex order parameter. r and ψ denote the amplitude and phase of the mean field. For values of $K < K_{cr}, r \approx 0$, resulting in an incoherent state, and for $K > K_{cr}, r \approx 1$, oscillators are synchronized, representing a coherent state.

III.2. Integrability and Passivity

Given a set of differential equations, one needs to find the existence and uniqueness of its solution. Picard's theorem states that if a function is continuous and Lipschitz, then a solution exists. If solutions exist, then how do we solve the equations to find solutions?. Many times, it may happen that solutions exist, but solving becomes problematic. Solvability implies a general formulation, implying that an algorithm can arrive at a solution for all the instances/parameter variations of the system. Such class of systems are said to be integrable and if generalized solutions cannot be arrived upon, systems are NI. If a system is integrable, one can explicitly solve the governing differential equations by the principle of first integrals to find the equilibrium point. We may assume that solutions exist and if system is integrable, then one can use either energy function (EF) or LF formulations to assess stability.

This is generally stated as the inverse problem in literature, implying, given a set of differential equations, could a Hamiltonian function, H, be formulated which assesses the stability of the system. The function H exists for conservative systems which are integrable. However, integrable systems are very rare in nature, most of the dynamical systems are NI and finding H for such systems is elusive, as the method of first integral fails here. Such systems can be split into systems which are integrable or passive and separate the non-passive terms. Controlling such systems would involve adding excess passivity to compensate for the shortage, through feedback. These systems are rendered as output feedback passive [9] and phase synchronization of such systems can be implemented through decentralized control, which requires adding excess passivity to overcome the deficit.

IV. Power System Stability Evaluation

Any physical system that is designed to perform in the steady state has to also operate satisfactorily within the desired margins during a sudden disturbance. When the physical systems are as complex and dynamically evolving as power systems, investigation of transient stability becomes crucial as the off-line studies may not be helpful [10] in predicting the unforeseen circumstances such as loss of generation or a critical transmission line. Using either linear or nonlinear models of the system, [11] determine feedback policies off-line that provide optimal control action through minimization of one or more cost functions.

The classical control laws [12], [13] may become ineffective with the increased complexity of the system, which is overcome by the model predictive control (MPC) strategy. The most challenging MPC application would be maintaining stability after large disturbances in a highly nonlinear, complex and hybrid system such as the power grid. For Single Machine Infinite Bus (SMIB), the nonlinear MPC with constraints on state and control variables, using tenth order linearized model of the power system, computes optimum value of TCSC reactance so as to control the first swing [14]. However, extending MPC for multi-machine systems proves difficult since TCSC reactance becomes a non-separable element of a higher order dense admittance matrix, and is not easily available as a control variable u; hence the need for better modeling approaches.

There are several techniques in literature which are universally accepted to analyze the transient behaviour of the system. Of them, since off-line Time Domain Simulation (TDS) are time consuming, a technique offered for this problem is the Lyapunov method [15]-[17] which directly assesses the degree of stability for a given configuration or an operating state. However, formulation of a Lyapunov function for a nonlinear system is a difficult task and EF is generally used as a Lyapunov function. Study of rotor angle stability of MMPS with TC has limitations in being extended beyond the two machine case, as the presence of TC makes the EF path dependent. Extending EF formulations for N machines with TC included is a challenging task as discussed in [18]. Borrowing idea from Watanabe Strogatz theory, this paper provides a proof of NI of N-machine system by considering the entire solution space of the dynamics of N-machines.

In order to give physical insights into the concept of frequency stability of power grids, [19] proposed the application of the phase synchronization of coupled oscillators to the rotor angle stability of MMPS, with the help of ideas from singular perturbation theory.

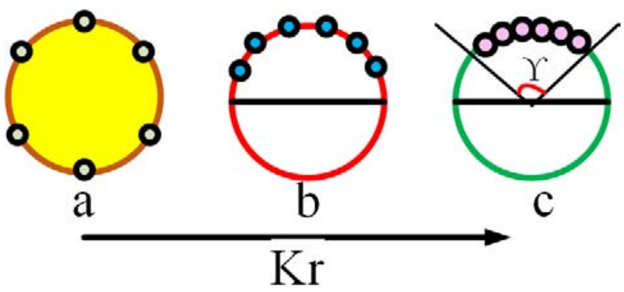
Frequency synchronization and phase cohesiveness are the conditions for achieving rotor angle stability. The geometric interpretations of these conditions are discussed in the next section.

IV.1. Importance of Convexity and How to Achieve It

Convexity is an important property which is used in solving multi-constrained problems. If the objective function is convex, then the optimization problem becomes simple to solve. The requirement of convexity imposed on rotor angles is equivalent to enforcing on the line flows, which can be reasoned out as follows: The amount of power transfer between generators is governed by the angular difference between the interconnecting buses. Therefore, it implies that the spread of distribution of line flows would imitate the spread of distribution of phase angles $\delta_i - \delta_j$. Hence, post fault, the control effort lies in framing a convex optimization problem [20] which will attain phase synchronization with rescheduling of line flows.

When agents interact with each other, they form a boundary and since machines are rotational bodies, the synchronization manifold is a circle. Since each machine is represented as an oscillator, the rotor angle is analogous to the phase of the oscillator thereby representing them as phase oscillators, with a solution space S^1 having rotational invariance. Figs. 3 show different plausible convex regions on the one dimensional torus which indicate different solutions of the Kuramoto model.

Fig. 3(a) shows one such stable state which is not convex with respect to the circumference, though the interior is. This configuration is achieved for zero coupling strength, wherein all oscillators operate at their natural frequencies. With increase in coupling strength $K > K_{cr}$, oscillators are in generating mode, thereby ensuring that they lie within the semicircle, as in Fig. 3(b). This geometric condition imposed on the generator rotor angles guarantees that convexity is achieved.



Figs. 3. Convex regions for oscillators

If all the generator angles lie within the sector of γ, i.e. $|\theta_i - \theta_j| < \gamma$, where $\gamma \in \{0, \pi/2\}$, then this is region corresponds to the region of positive invariance or where a positive invariant set exists, assuring phase synchronization. Fig. 4 shows the different regimes of synchrony and also the convex and non-convex regions for the phase oscillators. For the first order non-uniform Kuramoto model, as we move from the region $r = 0$ which corresponds to the incoherent regime, increase in K results in more generators contributing to the mean field, thereby resulting in an increase in r. When $K > K_{cr}$, an attractive region is established, rendering a convex conducive region for the phases of the oscillators.

With further increase in$K$ $(> K_{lock})$, all the phase angles move into the phase cohesive region which corresponds to the region of positive invariance. All the rotor angles should finally fall into this region so as to ensure phase synchronization.

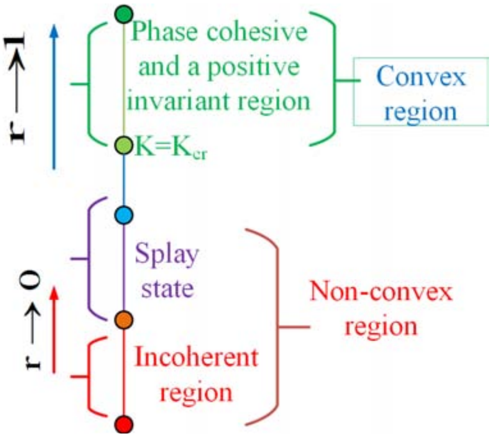


Fig. 4. Non-convex and convex regions

IV.2. Mechanisms of Achieving Rotor Angle Stability

Most often, synchronization phenomena arise in physical or biological systems where complex interactions in the coupled dynamics of the underlying physical processes cause the agents to synchronize [10].

At times, synchronization is explicitly sought, and appropriate control laws or algorithms are designed to drive the agents to synchrony. Different control strategies would result in attainment of different objectives, e.g., in the context of power grid, distributed control through mean field approach is central to accomplish synchronization of rotor angles. However, the decentralized control technique through individual LFs results in control to perturbations in the system which is a local approach. Apart from estimating the region of attraction (ROA), LF has also been used in forming the control law, the formulation being different depending majorly on the control strategy being imposed:

i) In the case of a fault which has occurred and the system moving away from synchrony, the law would modify the interconnection structure (Y_{bus}) so as to bring the trajectories within ROA, thereby establishing synchronization. This is achieved using distributed control and set-valued Lyapunov functions play a central role.

ii) However, a decentralized control strategy would adopt the passivity approach so as to formulate the LF for individual sub systems, treating the non-passive terms as perturbations, and the ensuing control law would enable to supply excess of passivity.

IV.3. Distributed Control Using Kuramoto Model

The assessment of the transient dynamics of the generators is represented through the swing equation:

$$\dot{\delta}_j = w_j - w_{ref}$$

$$M_j \dot{w}_j = -D_j w_j + \Omega_j - \sum_{k=1, k\neq j}^{N} \left[|E_k||E_j||Y_{kj}| \cos(\delta_k - \delta_i + \varphi_{kj}) \right] \quad (9)$$

Moving to the rotating reference frame, this equation is written in the second order form:

$$M_j \ddot{\theta}_j = -D_j \dot{\theta}_j + \Omega_j - \sum_{k=1, k\neq i}^{N} \left[P_{kj} \cos(\delta_k - \delta_i + \varphi_{kj}) \right] \quad (10)$$

For small inertia over damping ratio $\left(\frac{M_j}{D_j}\right)$ of generators, singular perturbation [19] separates slow and fast dynamics of the system:

$$\dot{\theta}_j = \Omega_j - \sum_{k=1}^{N} P_{kj} \sin(\theta_j - \theta_k + \beta_{kj}) \quad (11)$$

Equation (11) captures the power system dynamics sufficiently well during first swing. Synchronization of phases (θ_j), having different natural frequencies (Ω_j), are coupled through an all-to-all network with strengths (P_{kj}).

If $G_{kj} = 0$ ($\beta_{kj} = 0$), then the line is treated as lossless, else lossy. Post fault, the control action to achieve synchronization could either be by varying E_k, E_j (decentralized) or Y_{kj} (distributed). Changes in Y_{kj} can be achieved by inserting FACTS devices, which leads to modifying the P_{kj} in (11).

Eventually, this would result in change in line flows, thereby, resulting in phase angles in the convex region on the circle.

Therefore, the Kuramoto framework provides solutions to address stability of lossy MMPS and also provide strategies for control through the distributed control approach.

IV.4. Decentralized Control Using Passivity Approach

Stabilization of trajectories to reduce the differences between output variables through local information available with each agent, falls under the class of decentralized control. Achieving this task by developing feedback control laws greatly relies on notions of passivity. As the feedback interconnection of two passive systems results in a passive system, a systematic construction of LF in the form of sum of storage functions of individual sub-systems is possible [21]. As explained in Section III, for a lossless system, the Lyapunov function formed for the individual agents will ensure convergence to the equilibrium. However, the issue arises when losses are not negligible [22], and extending (1) for this case is not straightforward.

Such class of systems is analogous to the many-body problem of celestial mechanics. The Newtonian laws of motion have been primarily used to evaluate forces between interacting gravitational bodies. These laws were also applied to planetary masses resulting in differential equations which could be explicitly solved for two interacting bodies. However, for the three body case, analytical solutions fail to exist and such a class of systems falls under the genre of "open systems".

Remarkable observations revealed that the interactions between the planets can be neglected in comparison to that between the planet and the sun. This gives an integrable system, or one which can be solved explicitly, with each planet revolving around the sun oblivious of the others existence [23]. Series expansion methods have been used to evaluate the interactions between the planets through perturbations, though, establishing convergence of the series has proved to be futile.

A similar analogy can be applied to address stability of NI systems. Consider a general nonlinear dynamical system:

$$\dot{x} = f(x) + g(x)u \quad (12)$$

where $f(x)$ and $g(x)$ are smooth vector fields on $\mathcal{X} \in \Re^N$ which is the operating region of the system.

There could exist systems which fail to have $f(x)$, i.e. they are structurally NI and are known as non-holonomic systems, e.g., car or mechanical systems, the dynamics of which have constraints. Storage functions cannot be formed for such class of systems. However, for systems with $f(x)$, the vector field could be decomposed into an exact (passive) and a non-exact part (non-passive). A LF can be formed for the exact part, whereas the non-exact part is treated as a perturbation on the exact part, which can be controlled through damping. A procedure to generate potential function and to derive a control law has been discussed in [24]. Given any general dynamical system of the form (12), decompose $f(x)$ into its natural components, to get:

$$\dot{x} = \underbrace{f_d(x) + f_I(x)}_{passive} + \underbrace{f_{nd}(x)}_{non-passive} + g(x)u \quad (13)$$

In the absence of the non-passive term (losses), LF can be formulated to assess stability. Systems which are lossy fall under the genre of “output feedback passive” systems, wherein the dynamics are split into passive and non-passive parts, with the control strategy ensuring addition of excess of passivity to compensate for the shortage. This procedure has been validated for attaining rotor angle synchronization of N machine system with losses through excitation control [24]. In comparison to the control laws derived in [25], this method develops control formulations without solving PDEs.

V. Conclusion

Power grid security under critical cases of contingencies is an important study to ensure reliability and continuity of supply. A major contribution of this work is to view the power grid as an open system, which aids largely in throwing light on the challenges and solutions involved in its stability assessment. The notions of synchronization and stability and the context under which they are applicable to evaluate rotor angle stability of lossy MMPS is discussed. The paper highlights the two broad control architectures used to achieve rotor angle stability, the first being the mean field approach to synchronize rotor angles wherein convexity of the control law plays a central role. The second method emphasizes on the role of passivity in analyzing stability of NI systems, which finds applications in excitation control of the power grid.

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References

[1] W. Ren, R. W. Beard, E. M. Atkins, Information consensus in multivehicle cooperative control, *IEEE Control Systems, vol. 27*, no. 2, pp. 71–82, 2007

[2] A. Pikovsky, M. Rosenblum, and J. Kurths, *Synchronization: a universal concept in nonlinear sciences* (vol. 12, Cambridge university press, 2003).

[3] F. Blanchini and S. Miani, *Set-theoretic methods in control* (Springer, 2008).

[4] M. Zhu and S. Martínez, *Distributed Optimization-Based Control of Multi-Agent Networks in Complex Environments* (Springer, 2015).

[5] U. A. Khan, S. Kar, and J. M. Moura, Distributed average consensus: Beyond the realm of linearity, in *Signals, Systems and Computers, Conference Record of the Forty-Third Asilomar* pp. 1337–1342, IEEE, 2009

[6] L. Moreau, Stability of multiagent systems with time-dependent communication links, *IEEE Transactions on Automatic control*, vol. 50, no. 2, pp. 169–182, 2005

[7] P. DeLellis, M. di Bernardo, and G. Russo, On quad, lipschitz, and contracting vector fields forconsensus and synchronization of networks, *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 58, no. 3, pp. 576–583, 2011.

[8] M. di Bernardo, D. Fiore, G. Russo, and F. Scafuti, *Convergence, consensus and synchronization of complex networks via contraction theory* (Complex Systems and Networks, pp. 313–339, Springer, 2016.)

[9] H. K. Khalil, *Nonlinear Systems* (Prentice-Hall, New Jersey, 1996).

[10] S. C. Savulescu, *Real-time stability assessment in modern power system control centers* (vol. 42. John Wiley & Sons, 2009).

[11] L. Reznik, O. Ghanayem, and A. Bourmistrov, Pidplus fuzzy controller structures as a design base for industrial applications, *Engineering applications of artificial intelligence*, vol. 13, no.4, pp. 419–430, 2000.

[12] J. B. Burl, *Linear optimal control: H_2/H_∞ methods* (Addison-Wesley Longman Publishing Co., Inc., 1998).

[13] B. S. Chen, C. S. Tseng, and H.-J. Uang, MixedH_2/H_∞ fuzzy output feedback control design fornonlinear dynamic systems: an LMI approach, *IEEE Transactions on Fuzzy Systems*, vol. 8, no. 3, pp. 249–265, 2000

[14] Kulkarni, S., Wagh, S., Singh, N., Challenges in Model Predictive Control Application for Transient Stability Improvement Using TCSC, (2015) *International Review of Automatic Control (IREACO)*, 8 (2), pp. 163-169.

[15] T. Athay, R. Podmore, and S. Virmani, A practical method for thedirect analysis of transient stability, *IEEE Transactions on Power Apparatus and Systems*, no. 2, pp. 573–584, 1979

[16] A. Bose, Application of direct methods to transient stability analysis of power systems, *IEEE transactions on power apparatus and systems*, no. 7, pp. 1629–1636, 1984

[17] K. Padiyar and K. Ghosh, “Direct stability evaluation of power systems with detailed generator models using structure-preserving energy functions, *International Journal of Electrical Power & Energy Systems*, vol. 11, no. 1, pp. 47–56, 1989

[18] M. Parimi, S. Kulkarni, S. Wagh, K. Kumar, F.Kazi, and N. Singh, Synchronization andintegrability of N-machine system with transfer conductances, in *Power and Energy Society General Meeting (PESGM)*, 2016, pp. 1–5

[19] F. Dorfler and F. Bullo, Synchronization andtransient stability in power networks and nonuniform Kuramoto oscillators, *SIAM Journal on Control and Optimization*, vol. 50, no. 3, pp. 1616–1642, 2012

[20] S. Kulkarni, S. Wagh, F. Kazi, and N. Singh, Synchronization and transient stability in multi-machine power system through edge control using TCSC, in *Proceedings of the Mathematical Theory of Networks and Systems MTNS*, 2014

[21] M. Areak, Passivity as a design tool for group coordination, in *American Control Conference*, pp. 6, IEEE, 2006

[22] H. D. Chiang, *Direct methods for stability analysis of electric power systems: theoretical foundation, BCU methodologies, and applications* (John Wiley & Sons, 2011).

[23] C. E. Wayne, *An introduction to kam theory:Dynamical systems and probabilistic methods in partial differential equations* (Berkeley, 1994), pp. 3–29, 1996.

[24] S. Kulkarni, M. Parimi, S. Wagh, and N. Singh, Decomposition of drift vector field: An application to multi-machine transient stability enhancement, in *Power and Energy Society General Meeting (PESGM)*, 2016, pp. 1–5, IEEE, 2016.

[25] R. Ortega, M. Galaz, A. Astolfi, Y. Sun, and T. Shen, Transient stabilization of multimachinepower systems with nontrivial transfer conductances, *IEEE Transactions on Automatic Control*, vol. 50, no. 1, pp. 60–75, 2005.

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